PHYS 320 ANALYTICAL MECHANICS

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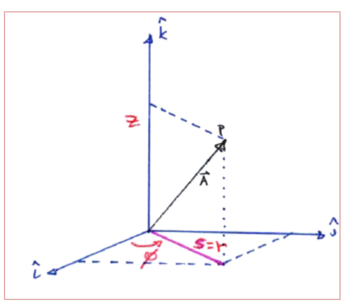
Cylindrical Coordinates

$r = s =$ POLAR COORDINATE – RADIAL (XY-PLANE)
$\phi =$ POLAR COORDINATE – ANGLE (X-Y-PLANE)
$z =$ CARTESIAN Z-COORDINATE

UNIT VECTORS: $\hat{r} = \hat{s}, \hat{\phi}, \hat{k}$

$x = r\cos\phi$
$y = r\sin\phi$
$z = z$

$\hat{r} = \hat{s} = \cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath}$
$\hat{\phi} = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}$
$\hat{k} = \hat{k}$



$s = \sqrt{x^2 + y^2}$
$\phi = \tan^{-1}[y/x]$
$z = z$

$\hat{\imath} = \cos\phi\,\hat{r} - \sin\phi\,\hat{\phi}$
$\hat{\jmath} = \sin\phi\,\hat{r} + \cos\phi\,\hat{\phi}$
$\hat{k} = \hat{k}$

$d\vec{\ell} = dr\,\hat{r} + r\,d\phi\,\hat{\phi} + dz\,\hat{k}$
$d\tau = dV = r\,dr\,d\phi\,dz$
$d\vec{A}$ DEPENDS ON SURFACE!

DIV, GRAD, and CURL are no longer so simple!!

Driven Oscillations (linear damping)

- Damped harmonic oscillator driven by time dependent external force:

$$m\ddot{x} + c\dot{x} + kx = F_{ext}(t)$$

inhomogeneous!

- Look at simple, periodic driving force:

$$F_{ext}(t) = \mathrm{F}_o \cos(\omega t) = \mathrm{Re}\{\mathrm{F}_o \mathrm{e}^{i\omega t}\}$$

it will be simpler to deal with complex numbers

- Solution to differential equation will be the sum of two parts:

$$x(t) = x_c(t) + x_p(t)$$

complementary soln.
(soln to homogeneous eqn)
transient term
{we know this already!}

"particular integral"
(soln to inhomogeneous bit)
steady-state term

Driven Oscillations (linear damping)

$$x(t) = x_c(t) + x_p(t)$$

$$x_c(t) = C_+ e^{-\gamma t} e^{+i\omega_d t} + C_- e^{-\gamma t} e^{-i\omega_d t} = Ae^{-\gamma t}\cos(\omega_d t - \delta)$$

$x_p(t) = ??$ ⇒ how about a trial solution of $x_p(t) = Ae^{i(\omega t - \delta)}$?

- This works if .....

$$\delta = \tan^{-1}\left(\frac{2\gamma\omega}{\omega_o^2 - \omega^2}\right) = \tan^{-1}\left(\frac{c\,\omega}{k - m\,\omega^2}\right)$$

$$A = \frac{F_o}{\left[(k - m\,\omega^2)^2 + c^2\,\omega^2\right]^{1/2}} = \frac{F_o / m}{\left[(\omega_o^2 - \omega^2)^2 + 4\gamma^2\,\omega^2\right]^{1/2}} = \frac{F_o}{m\,D(\omega)}$$

Driven Oscillations (linear damping)

$$x(t) = x_c(t) + x_p(t) = C_+ e^{-\gamma t} e^{+i\omega_d t} + C_- e^{-\gamma t} e^{-i\omega_d t} + A e^{i(\omega t - \delta)}$$

or

$$x(t) = x_c(t) + x_p(t) = A_d e^{-\gamma t} \cos(\omega_d t - \phi) + A\cos(\omega t - \delta)$$

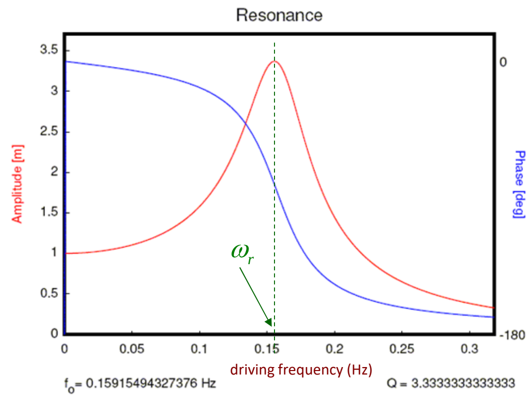
driving frequency

where

$$\delta = \tan^{-1}\left(\frac{2\gamma\omega}{\omega_o^2 - \omega^2}\right)$$

$$A = \quad = \frac{F_o / m}{\left[(\omega_o^2 - \omega^2)^2 + 4\gamma^2\omega^2\right]^{1/2}}$$

$$\omega_r^2 \equiv \omega_o^2 - 2\gamma^2$$



Fourier Series

For a function defined on the interval [-L, L], where

$$f(x') = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x'}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x'}{L}\right).$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x')\,dx'$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x')\cos\left(\frac{n\pi x'}{L}\right)dx'$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x')\sin\left(\frac{n\pi x'}{L}\right)dx'.$$

Similarly, the function is instead defined on the interval $[0, 2L]$, the above equations simply become

$$a_0 = \frac{1}{L}\int_{0}^{2L} f(x')\,dx'$$

$$a_n = \frac{1}{L}\int_{0}^{2L} f(x')\cos\left(\frac{n\pi x'}{L}\right)dx'$$

$$b_n = \frac{1}{L}\int_{0}^{2L} f(x')\sin\left(\frac{n\pi x'}{L}\right)dx'.$$